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Decision Support

Nash marginal abatement cost estimation of air pollutant emissions using the stochastic semi-nonparametric frontier

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ABSTRACT

Emissions trading (or cap and trade) is a market-based approach providing economic incentives for achieving reductions in the emissions of pollutants. Marginal abatement costs (MAC), also termed shadow prices of air pollution emissions, provide valuable guidelines to support environmental regulatory policies for CO_2, SO_2 and NO_x, the key contributors to climate change, smog, and acid rain. This study estimates the marginal abatement cost of undesirable outputs with respect to the Nash equilibrium on the stochastic semi-nonparametric envelopment of data (StoNED) in an imperfectly competitive market. Considering an endogenous price function of electricity, the mixed complementarity problem (MiCP) is formulated to identify the Nash equilibrium in a production possibility set. The proposed model addresses the four issues of MAC estimation in the existing literature. Applying the proposed method to an empirical study of 33 coal-fired power plants operating in China in 2013 shows that StoNED provides a robust frontier that is not sensitive to the outlier and the proposed interval of MAC estimation validates the shadow prices corresponding to the Nash equilibrium in an imperfectly competitive market.

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1. Introduction

Burning coal to generate electricity produces carbon dioxide (CO_2), sulfur dioxide (SO_2), and nitrogen oxide (NO_x), the major causes of greenhouse effects, acid rain, and smog. Two decades ago, Greenhouse Development Rights (GDRs) were developed to provide a framework for sharing the costs of rapid climate change among countries (Berk & den Elzen, 2001). Based on scientific and political understandings of GDR, the framework contributed to the foundation of the United Nations Framework Convention on Climate Change (UNFCCC) in 1992. In 1997, the Kyoto Protocol adopted to link UNFCCC established legally binding obligations for developed countries to reduce their greenhouse gas emissions, e.g., the US agreed to reduce its emissions by 7% of the base year 1990 by 2012. Then, the 2012 Doha climate change conference focused on new carbon targets and a second commitment period of emissions reductions from 2013 to 2020. Later on, in December of 2015 in Paris conference, 187 countries adopted a historic international agreement on addressing global climate change under the UNFCCC that their governments agreed to a target of peaking the global greenhouse gas emissions as soon as possible so as to keep the increase of global average temperature below 2 °C and to pursuing effort to limit it to 1.5 °C above preindustrial levels. In 2015, these countries submitted their Intended Nationally Determined Contributions (INDCs) to the UNFCCC, outlining their intended post 2020 climate action plans (den Elzen et al., 2016a).

Meanwhile, the global economics community was debating market approaches to environmental externalities, including emissions taxes and permit trading systems. The market-based approach called cap-and-trade provides economic incentives for achieving emissions reductions. The basic concept places a cap on the number of emission allowances assigned to emission sources such as power plants, factories, etc. For example, each allowance gives the holder the right to emit one ton of CO_2, the main greenhouse gas, or the equivalent amount of pollutants. If the emissions reduction fails to meet the targets, the holder can participate in a permit trading market; that is, the marginal abatement costs (MAC) (or shadow prices) of pollutants represent the costs of reducing one extra unit of pollutant. The emissions trading mechanism is based on the seminal study regarding the Coase theorem (Coase, 1960). Coase (1960) argued that without a transaction cost, the bargaining would lead to an efficient outcome regardless of the initial allocation of property rights if trading in an externality. Later, Dales (1968) proposed the conceptual model of an emission trading market for pollutants.

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This study focuses on estimating MACs of coal-fired power plants in China area. China continues to be one of the world's major CO_2 emitters. Since 2007, China has become the world's largest contributor to carbon emissions from fossil fuel burning and cement production, and responsible for approximate 25% of global carbon emissions; its manufacturing and power generation sectors accounted for 85% of China's total carbon emissions (Liu, 2015). From 2013 to 2015, most of the provinces routinely experienced smog with high concentrations of PM 2.5. A vast area from Xi'an in central China to Harbin in the northeast was also badly affected. For example, on November 8, 2015, Liaoning province reached 1400 micrograms per cubic meter, which is 56 times the maximum safe level defined by the World Health Organization (WHO). A month later, China issued its first-ever "red alert" due to high levels of air pollution; in particular, the two key components of urban smog and acid rain are emissions of SO_2 and NO_x (Zhang & Samet, 2015).

In 2009, China claimed it would decrease its CO_2 emissions per unit of GDP (i.e., carbon intensity) by 40–45% by 2020 using 2005 as the reference year, and then set a target to reduce carbon intensity by 17% by 2015 compared to 2010. Since 2013, seven pilot provinces and provincial cities, Shenzhen, Shanghai, Beijing, Guangdong, Tianjin, Chongqing, and Hubei, successively launched their own emission trading schemes (ETS). These regional pilot carbon markets are regarded as indispensable experiments prior to establishing a national ETS. In 2015, before the Paris conference, China submitted its INDC which includes three major elements regarding emissions: (i) to peak CO_2 emissions no later than 2030, (ii) to increase the share of non-fossil fuels in the total primary energy supply to 20% by 2030, and (iii) to reduce the carbon intensity of GDP by 60-65% compared to 2005 levels by 2030 (den Elzen et al., 2016b).

A deeper understanding of MACs helps the decision-making process. For the policy-makers, China's regional government can formulate appropriate carbon abatement policies, such as using the estimated MACs as a reference for carbon pricing through emission allowance allocations, returning the price of allowances to its real value, increasing the penalty for parties that do not hand over the allowances in time, auctioning the allowances at a price higher than the market price, and allowing new external investors to trade allowances. For the firm managers, by comparing the market prices of the allowances with MAC, they can make more reasonable business decisions by screening a list of candidate policy options, e.g., business as usual, carbon abatement by technological investment, purchasing allowances, selling allowances, and reserving redundant allowances for future needs (Lee, Park, & Kim, 2002; Zhou, Fan, & Zhou, 2015). Thus, a firm can use more costly but cleaner green technology or a less costly but more polluting regular technology, or both (Gong & Zhou, 2013). To examine China's options, this paper uses the North and the Northeast regions as a case for estimating the MACs of CO_2, SO_2, and NO_x emissions from China's coal-fired power plants in 2013 due to a high concentration of PM2.5 in the two regions (Yang, 2014).

Many previous studies have estimated the shadow prices of pollutants or undesirable outputs. Parametric and nonparametric approaches were developed to estimate shadow prices. In fact, shadow price is a differentiable characteristic of production function. Parametric approaches are commonly used because a specified production function is differentiable everywhere. Färe, Grosskopf, Lovell, and Yaisawarng (1993) employed an output distance function with the translog functional form and used linear programming to solve for the combination of parameters that yielded the best-fit distance function to estimate a shadow price of four pollutants generated by pulp and paper mills operating in Michigan and Wisconsin in 1976. Coggins and Swinton (1996), who took the same approach to estimate the SO_2 shadow price of Wisconsin coal-burning utility plants in 1990–1992, showed that the average shadow price was above some observed allowance prices in the trading market. Färe, Grosskopf, Noh, and Weber (2005) used a quadratic directional output distance function to estimate both technical efficiency and a shadow price of SO_2 for U.S. electric utilities in 1993 and 1997. The result showed that emissions trading became difficult since the shadow price of SO_2 increased from 1993 to 1997. They also measured the output elasticity of substitution between electricity and SO_2 and found that inefficiency reduction (i.e., productivity improvement) benefited the emissions reduction. Marklund and Samakovlis (2007) evaluated the Burden-Sharing Agreement (BSA) which redistributes the emission reduction target in European Union (EU) under 8% reduction of 6 greenhouse gases during 2008–2012 described in the Kyoto Protocol, and used the directional output distance function to estimate MAC based on the production data of EU member states for 1990–2000.

Nonparametric methods (e.g., data envelopment analysis, DEA) can estimate the production function without a specified functional form. A directional distance function is commonly used to investigate the shadow price on the efficient frontier. Boyd, Molburg, and Prince (1996) used DEA to estimate the efficient frontier and the MAC of SO_2 for 29 coal-burning utilities in the U.S. electric power industry. Finding some positive shadow prices against intuition, the authors claimed it was due to the lack of uniformity in environmental regulations in 1989. Nagurney and Dhanda (2000) modeled systems of marketable pollution permits using a variational inequality approach, which explicitly handled spatial differentiation for the pollutants as well as the pollutants transferred between different firms. The approach ensured that the imposed environmental quality standards were met through the initial allocation of licenses, but did not take the production function and its differential characteristics into account, and thus the capacity level and marginal productivity were not appropriately formulated. Lee et al. (2002) showed that the shadow price of a pollutant was the product of the inefficiency correction factor and the slope to the frontier, where the inefficiency correction factor was the inefficiency ratio between undesirable output and desirable output, which mapped an inefficient point to the corresponding point on the production frontier. They accounted for the technical inefficiency to derive the shadow prices of SO_2, NO_x and total suspend particulates (TSP) for Korean coal- and oil-burning power plants from 1990–1995. However, the DEA estimator was sensitive to outliers and shadow price values equal to zero were common; in particular, DEA did not consider statistical noise and assumed the observations were assessed without error. To address this issue, Kuosmanen (2008) proposed convex nonparametric least squares (CNLS) approach which incorporated the merits of parametric and nonparametric approaches. Kuosmanen and Johnson (2010) showed that DEA was a special case of CNLS with sign constraints on error terms. Mekaroonreung and Johnson (2012) used CNLS approach in two ways – with random noise and without random noise – to estimate the shadow prices of SO_2 and NO_x in U.S. bituminous coal power plant boilers. The result showed that both SO_2 and NO_x shadow prices estimated by CNLS and accounting for noise were in reasonable ranges and comparable to the allowance market prices. However, previous studies only considered an exogenous price of electricity and assumed a perfectly competitive market.

This paper addresses four issues in the existing literature of MAC estimation and proposes the MAC of the Nash equilibrium benchmark (Nash-MAC hereafter) on the stochastic semi-nonparametric envelopment of data (StoNED) frontier in an imperfectly competitive market with an endogenous price. First, since the MAC (or shadow price) is the differential characteristic of the production frontier, most studies estimate the

MAC by projecting the inefficient firm to the frontier given a pre-defined direction (i.e., orientation) (Lee et al., 2002). The different directions assigned, however, may generate significantly different shadow price estimations, and thus there is no best way to determine the direction (Boyd et al., 1996; Coggins and Swinton, 1996; Färe, Grosskopf, & Whittaker, 2013; Lee et al., 2002; Turner, 1995; Vardanyan & Noh, 2006). Typical studies of productivity and efficiency analysis implicitly assume a perfectly competitive market and an exogenous market price. In practice, however, the energy market is an imperfectly competitive market and the market price can be affected by the total supply generated by all firms in the market (Hobbs & Pang, 2007), i.e., the market price is endogenous and all firms are not independent from each other. Therefore, we consider the inverse demand function (i.e., price function) in an imperfectly competitive power market to calculate the Nash equilibrium regarded as an efficient benchmark under a competitive production behavior, and thus projecting the inefficient firm toward a Nash solution has important economic implications. Note that the market price of electricity depends on both demand and supply sides, and this study suggests the inverse demand function considering the demand side and the estimated production function restricting the electricity-generating behavior considers the supply side (Gabriel, Conejo, Fuller, Hobbs, & Ruiz, 2013; Hobbs & Pang, 2007; Yang, Chung, & Wong, 2012).

Second, since each firm has its specific benchmark due to predefined orientation, these benchmarks generate different MACs. If the firm-specific MACs differ significantly due to the projecting orientations, then the average value could be affected by outliers and give a biased result. We suggest the Nash equilibrium as the only one efficient benchmark for all the firms and thus there is no need to calculate the average or weighted average of the different MACs as the final value.

Third, Stochastic Frontier Analysis (SFA) and Data Envelopment Analysis (DEA) are the two most common ways to estimate a frontier production function. But SFA is a parametric regression-based method that requires a prior specification of the functional form of the frontier, whereas DEA is a nonparametric linear programming approach that avoids the functional form assumption and no assumption of the inefficiency distribution, but also assumes a deterministic frontier without considering noise and attributes all deviations from the frontier to inefficiency. In particular, the DEA estimator is sensitive to outliers and the shadow price values equal to zero are quite common (Mekaroonreung & Johnson, 2012). We alternatively estimate the Nash equilibrium on the StoNED frontier, because it integrates the merits of both SFA and DEA, which avoids the functional form assumption, builds on the same axioms as DEA, and accounts for noise (Kuosmanen, 2008; Kuosmanen & Johnson, 2010; Kuosmanen & Kortelainen, 2012; Lee, Johnson, Moreno-Centeno, & Kuosmanen, 2013).

Fourth, many studies estimate the shadow prices of multiple undesirable outputs separately. For instance, estimating the shadow price of SO_2 independently from estimating the shadow price of NO_x based on the same production function assumes implicitly that a firm can only generate one type of pollutant at a time. Estimating the marginal product of pollutants individually, however, may cause an overestimation of the marginal product of one pollutant and an underestimation of its shadow price (Lee & Zhou, 2015). Thus, we adopt the directional marginal productivity (DMP) technique (Lee, 2014) to calculate the directional MACs of CO_2, SO_2, and NO_x emissions simultaneously.

For comparison, Table 1 summarizes previous studies, where each cases use different datasets, regions, and estimation methods.

The remainder of this paper is organized as follows. Section 2 introduces the Nash equilibrium solution solved by a mixed complementarity problem (MiCP) with respect to the StoNED frontier and then estimates the Nash-MACs of multiple undesirable outputs via the DMP technique explained in Section 3. Section 4 describes an empirical study of China's coal power industry to estimate the Nash-MACs of CO_2, SO_2, and NO_x. The results are compared with previous studies to provide managerial insights. Section 5 concludes.

2. Nash equilibrium identified on the StoNED frontier

This section describes the stochastic frontier called stochastic semi-nonparametric envelopment of data (StoNED) (Kuosmanen & Kortelainen, 2012; Kuosmanen, Johnson, & Saastamoinen, 2015) and identifies the Nash equilibrium on the frontier with inputs, desirable outputs, and undesirable outputs.

Nash equilibrium is a solution of a non-cooperative game involving two or more players, and no player has incentive to change the strategy due to a reduction in the immediate payoff (Nash, 1951). A strategy is a complete plan of actions, specifying the move that a player would make at each information set which establishes all the possible moves that could have taken place in the game so far, given what that player has observed. This study focuses on the perfect information (i.e. players observe each other's moves well) and one-shot statistic game (i.e. players interacts with simultaneous moves). For details of Nash equilibrium identified in DEA framework, see Lee and Johnson (2015). Lee (2016) extended it to measure of market structure, and Lee (2018) proposed the mixed-strategy Nash equilibrium in DEA.

For StoNED framework, it is a semi-parametric regression technique that considers the noise and inefficiency and does not specify the functional form *a priori* while maintaining the standard regularity conditions from microeconomic theory for production functions, namely continuity, monotonicity, and concavity. To build the StoNED frontier, convex nonparametric least squares (CNLS) was developed (Hildreth, 1954; Kuosmanen, 2008; Kuosmanen & Johnson, 2010) and followed by integrating the noise term into the DEA framework referring to StoNED.

2.1. Convex nonparametric least squares (CNLS)

Consider a regression function, $y = f(\boldsymbol{x}) + \varepsilon$, with shape restrictions that is estimated via CNLS, where y is the dependent variable, $\boldsymbol{x}$ is a vector of independent (explanatory) variables, and ε is a composite error equal to noise v plus asymmetric inefficiency u, i.e., $\varepsilon = v - u$. In particular, the zero-mean assumption is violated, i.e., $E(\varepsilon) = E(v - u) = -E(u) < 0$. Assume that regression function $f(\cdot)$ satisfies the monotonicity and concavity. If the inefficiency term u has a constant variance, i.e., inefficiency term u is homoscedastic, then the expected value of the inefficiency term u is a constant, denoted as μ. In other words, the CNLS provides a consistent estimator of the frontier f minus a constant (Kuosmanen & Kortelainen, 2012).

Let index $i \in I$ represent the input, index $q \in Q$ the undesirable output, and index $k \in K$ the set of decision making units (DMU) or firm index. Observations X_{ik} represent the ith input level, Y_k the single desirable output (i.e. electricity generation) level, and B_{qk} the qth undesirable output level of firm k. Formulate CNLS as a quadratic programming to estimate the production function as:

$$\min_{\alpha,\beta,\gamma,\varepsilon} \sum_{k\in K} \varepsilon_k^2$$

$$\text{s.t. } Y_k = \alpha_k + \sum_{i\in I} \beta_{ik} X_{ik} + \sum_{q\in Q} \gamma_{qk} B_{qk} + \varepsilon_k \;\forall\, k \in K$$

$$\alpha_k + \sum_{i\in I} \beta_{ik} X_{ik} + \sum_{q\in Q} \gamma_{qk} B_{qk} \le \alpha_h + \sum_{i\in I} \beta_{ih} X_{ik} + \sum_{q\in Q} \gamma_{qh} B_{qk} \;\forall\, k,\; h \in K \text{ and } k \ne h$$

Table 1
Studies for MAC estimations in electric power sectors (extended from Lee & Zhou, 2015).

Study	Country	Year	Level	Sample size	Price of electricity (US$/megawatt hour)	Frontier estimation	Direction of inefficient unit projected to frontier[a]
Atkinson and Dorfman (2005)	U.S.	1980, 1985, 1990, 1995	Firm	43	Not applicable	Translog	$g^Y > 0$
Boyd et al. (1996)	U.S.	1989	Plant	62	50.00	DEA	$g^Y > 0, g^B < 0$
Coggins and Swinton (1996)	U.S.	1990–1992	Plant	42	36.38–65.87	Translog	$g^Y > 0, g^B > 0$
Färe et al. (2005)	U.S.	1993, 1997	Boiler	209	10.39–100.42	Quadratic DDF	$g^Y > 0, g^B < 0$
Gupta (2006)	India	1990–2000	Plant	76	Not available	Translog	$g^Y > 0, g^B > 0$
Harkness (2006)	U.S.	2000	Plant	518	51.58–52.96	Translog	$g^Y > 0, g^B > 0$
Lee et al. (2002)	Korea	1990-1995	Plant	43	66.67	DEA	$g^Y < 0, g^B < 0$
Matsushita and Yamane (2012)	Japan	2000–2009	Firm	76	161.41	Quadratic DDF	$g^Y > 0, g^B < 0$
Mekaroonreung and Johnson (2012)	U.S.	2000–2008	Boiler	336	17.26–165.70	CNLS	$g^Y > 0$
Park and Lim (2009)	Korea	2001–2004	Plant	80	56.07	Translog	$g^Y > 0, g^B > 0$
Rezek and Campbell (2007)	U.S.	1998	Plant	260	41.6–119	Translog	$g^Y > 0$
Turner (1995)	U.S.	1985–1987	Plant	147	32.98–93.47	DEA	$g^Y > 0$
Zhou et al. (2015)	China	2009–2011	Sector	30	Not applicable	DEA	$g^Y > 0, g^B < 0$
Lee and Zhou (2015)	U.S.	1990–2010	State	48	33.7–180.6	DEA	Only efficient unit
Present study	China	2013	Plant	33	Endogenous price 71.74	StoNED DEA	Only NPF or PF benchmark

[a] $g^B < 0$ indicates the emission reduction direction.

$$\alpha_k + \sum_{i\in I} \beta_{ik} X_{ih} \geq 0 \quad \forall\, k,\ h \in K \text{ and } k \neq h$$
$$\beta_{ik},\ \gamma_{qk} \geq 0 \quad \forall\, k \in K,\ i \in I,\ q \in Q \tag{1}$$

where ε_k is the composite error that represents the deviation of firm k from the estimated function and decision variables α_k, β_{ik}, and γ_{qk} characterize the intercept and slope parameters regarding the marginal products of the inputs and undesirable outputs for each observation. The objective function minimizes the sum of the square with respect to the disturbance terms. The first equality constraints represent a basic linear regression formulation. The second inequality constraints impose concavity using Afriat's inequalities (Afriat, 1972). The third inequality constraints impose the weak disposability between the desirable and undesirable outputs, for which reducing the desirable output will result in a proportionate reduction of the undesirable outputs (Fried, Lovell, & Schmidt, 2008; Shephard, 1974). The fourth constraints impose monotonicity of both the inputs and the costs associated with additional undesirable outputs on the underlying unknown function. Because the second constraint can generate a large number of constraints and impose a computational burden when considering a large amount of observations, Lee et al. (2013) proposed an efficient algorithm for CNLS to address the large-scale problem size.[1]

Given the CNLS residual $\hat{\varepsilon}_k$ obtained from (1), estimating the expected value of inefficiency, $\mu = E(u_k)$, by the method of moments (Aigner, Lovell, & Schmidt, 1977) shifts the CNLS frontier (i.e., the lower bounds of concave envelope) upward by adding the expected inefficiency to estimate the StoNED frontier. We assume half-normal inefficiency $u_k \sim N^+(0, \sigma_u^2)$ and normal noise $v_k \sim N(0, \sigma_v^2)$, since the method of moments requires some additional parametric distributional assumptions. The second central moment of the residual distribution is $\hat{M}_2 = \sum_{k=1}^{|K|} (\hat{\varepsilon}_k)^2/(|K|-1)$. The third central moment of the residual distribution is $\hat{M}_3 = \sum_{k=1}^{|K|} (\hat{\varepsilon}_k)^3/(|K|-1)$. The hat on top of the third central moment indicates the true estimator but unknown values of the central moments. In fact, based on half-normal inefficiency assumption, the third central moment is equal to $M_3 = (\sqrt{\frac{2}{\pi}})(1-\frac{4}{\pi})\sigma_u^3$. Thus, given the estimated $\hat{M}_3$, estimate σ_u as $\hat{\sigma}_u = \sqrt[3]{\frac{\hat{M}_3}{(\sqrt{\frac{2}{\pi}})(1-\frac{4}{\pi})}}$. Finally, estimate the expected value of inefficiency as $\hat{\mu} = \hat{\sigma}_u\sqrt{\frac{2}{\pi}}$, and thus the StoNED frontier is the CNLS frontier obtained from model (1) plus estimated expected inefficiency $\hat{\mu}$.

Note that, for more diverse applications, the convex regression estimated by CNLS could be replaced by segmented convex regression (Keshvari, 2018), convex adaptive partitioning (CAP) (Hannah & Dunson, 2013), or shape constrained kernel-weighted least squares (SCKLS) (Yagi, Chen, Johnson, & Kuosmanen, 2018). Method of moments could be replaced by pseudolikelihood (Fan, Li, & Weersink, 1996) or kernel deconvolution (Hall & Simar, 2002).

2.2. Profit maximization on StoNED frontier

For modeling the profit-maximizing behavior in the imperfectly competitive market. Given index r representing one specific firm and an alias of index k, let x_{ir}, y_r, and b_{qr} represent the decision variables for input i, single desirable output, and undesirable output q of one specific firm r. To build a price function of the desirable output, consider an inverse demand function as $P^Y(\hat{Y}) = P^{Y0} - \kappa\hat{Y}$, where $P^Y(\cdot) \geq 0, \hat{Y} = (\sum_k y_k + \bar{Y}), \bar{Y}$ is a constant representing the least and fixed output levels generated by the plants without market power, P^{Y0} is a positive intercept, and $\kappa \geq 0$ indicates the price sensitive coefficient of the desirable output.[2]

Apparently, the revenue function $P^Y(\hat{Y})y_r$ is concave (Lee & Johnson, 2015; Murphy, Sherali, & Soyster, 1982). For the undesirable output of the power market, i.e., environmental externalities, set a legal limit (cap) on the quantity of a certain type of emission an economy can emit in a period, to control emissions. We limit the feasible region of the undesirable quantity $\hat{B}_q \leq \hat{B}_q^{CAP}$, where $\hat{B}_q = \sum_k b_{qk}$ and $\hat{B}_q^{CAP}$ is a constant representing the limit of total emissions. Without loss of generality, for the input of the power market, i.e., coal consumption, we assume a competitive input market and the price of input is a constant, P_i^X. Given $X_i^{max} = \max\{X_{ik}\}$, the input scale constraint $x_{ir} \leq X_i^{max}$ indicates that the Nash equilibrium can be a feasible efficient benchmark presenting the bounded input scale.[3]

$\in$ is a small positive constant. Given the optimal coefficients $\hat{\alpha}_k$, $\hat{\beta}_{ik}$, and $\hat{\gamma}_{qk}$ obtained from model (1) and the cap constraint $\hat{B}_q \leq \hat{B}_q^{CAP}$ of the undesirable output, we define our maximization model

[1] Similar to DEA generating non-unique shadow prices, formulation (1) generates multiple solutions. However, the issue does not affect the shape of the production function and its differential characteristic, i.e., MAC, investigated in this paper. For a unique solution, see Kuosmanen and Kortelainen (2012).

[2] This study investigates the production behavior of the plants with nameplate capacity higher than 1000 megawatts due to market power, and thus $\sum_k y_k$ indicates the total output from these plants.

[3] The input scale constraint $x_{ir} \leq X_i^{max}$ ensure a bounded input in the production possibility set (PPS) and guarantee the existence of the Nash solution. See Lemma 2.2 and Theorem 2.1.

of Nash profit function (NPF) with respect to the StoNED frontier as in the following equation:

$$NPE_r^{StoNED*} = \max_{y_r, b_{qr}, x_{ir}} \left\{ (P^{Y_o} - \kappa\hat{Y})y_r - \sum_{i\in I} P_i^X x_{ir} - \epsilon \sum_{q\in Q} b_{qr} \;\middle|\; \begin{array}{c} y_r \le \hat{\alpha}_k + \sum_{i\in I}\hat{\beta}_{ik}x_{ir} + \sum_{q\in Q}\hat{\gamma}_{qk}b_{qr} + \hat{\mu}, \forall k \in K \\ \hat{\alpha}_k + \sum_{i\in I}\hat{\beta}_{ik}x_{ir} + \hat{\mu} \ge 0, \forall k \in K \\ \hat{B}_q \le \hat{B}_q^{CAP}, \forall q \in Q; \\ x_{ir} \le X_i^{\max}, \forall i \in I \\ x_{ir}, y_r, b_{qr} \ge 0, \forall i \in I, q \in Q \end{array} \right\} \quad (2)$$

In previous studies, the Nash solution exists and is unique if the profit function is strictly concave and the feasible region is a finite convex set (Facchinei & Pang, 2003; Hobbs & Pang, 2007). Lee and Johnson (2015) proved the existence of the Nash solution in the production possibility set (PPS) estimated by DEA frontier given a linear inverse demand function of desirable outputs. Extending to the undesirable output case of model (2), the objective function $(P^{Y_0} - \kappa\hat{Y})y_r - \sum_{i\in I} P_i^X x_{ir} - \epsilon \sum_{q\in Q} b_{qr}$ is strictly concave on a convex PPS enveloped by the StoNED frontier, and it verifies the existence and uniqueness of the Nash solution.[4] The term $\sum_{q\in Q} b_{qr}$ close to zero implies that a firm's productive behavior depends on physical profit maximization and the regulation of pollutant emission is only a requirement rather than an objective function. This assumption conforms with business practice. Note that even if model (1) generates multiple optimal solutions, it does not affect the result of model (2), since the output level estimated by equation $\hat{\alpha}_k + \sum_{i\in I}\hat{\beta}_{ik}x_{ir} + \sum_{q\in Q}\hat{\gamma}_{qk}b_{qr} + \hat{\mu}$ is fixed to represent the frontier. In addition, if the price of electricity is exogenous with average price P^{Y_0} and $\kappa = 0$, model (2) will present a typical profit function and provide a profit maximization benchmark (Nerlove, 1965; Zofio, Paster, & Aparicio, 2013). We derive Lemma 2.1 to define the Nash equilibrium with both desirable and undesirable outputs.

Lemma 2.1. *Consider an imperfectly competitive market with $|K|$ firms, an inverse demand function $P^Y(\cdot)$ that is nonincreasing and continuously differentiable in y_k, and an inverse supply function $P^X(\cdot)$ that is nondecreasing (or in our case, a constant) and continuously differentiable in x_k. If the profit function $\pi_k(x_k, y_k, b_k)$ is concave and the variables $x_k, y_k, b_k \ge 0$, then $(\boldsymbol{x}^*, \boldsymbol{y}^*, \boldsymbol{b}^*) = ((\boldsymbol{x}_1^*, \boldsymbol{y}_1^*, \boldsymbol{b}_1^*), (\boldsymbol{x}_2^*, \boldsymbol{y}_2^*, \boldsymbol{b}_2^*), \ldots, (\boldsymbol{x}_{|K|}^*, \boldsymbol{y}_{|K|}^*, \boldsymbol{b}_{|K|}^*))$ is a Nash equilibrium solution if and only if*

$$\nabla_{x_k}\pi_k(\boldsymbol{x}^*, \boldsymbol{y}^*, \boldsymbol{b}^*) \le 0 \text{ and } \nabla_{y_k}\pi_k(\boldsymbol{x}^*, \boldsymbol{y}^*, \boldsymbol{b}^*) \le 0, \forall k;$$
$$\boldsymbol{x}_k^*\left[\nabla_{x_k}\pi_k(\boldsymbol{x}^*, \boldsymbol{y}^*, \boldsymbol{b}^*)\right]0 \text{ and } y_k^*\left[\nabla_{y_k}\pi_k(\boldsymbol{x}^*, \boldsymbol{y}^*, \boldsymbol{b}^*)\right],$$

where $(\boldsymbol{x}_k^*, \boldsymbol{y}_k^*, \boldsymbol{b}_k^*) \in \tilde{T}$ and $\tilde{T}$ is the PPS estimated by the constraints of model (2).

Proof. See Appendix.

Lemma 2.2. *Based on Theorem 3.2 in Kuosmanen and Kortelainen (2012) and Lee and Johnson (2015), the convex polyhedral PPS estimated by DEA directly carries over to StoNED as the corresponding convex polyhedron PPS. Thus, model (2) provides a closed and convex PPS restricted by the StoNED frontier with cap constraint.*

Proof. See Appendix.

[4] Note that the NPF only considers the price of desirable output and the price of inputs since the price (i.e., MAC) of the undesirable output is the unknown variable we would like to estimate. However, if setting the objective function $(P^{Y_0} - \kappa\hat{Y})y_r - \sum_{i\in I} P_i^X x_{ir}$ in model (2), the Nash solution may not be unique. Thus, we suggest adding the term $-\sum_{q\in Q} b_{qr}$ in the objective function for the solution uniqueness.

To solve for a Nash equilibrium in Eq. (2), define the Lagrangian function as:

$$\begin{aligned} &L_r(x_{ir}, y_r, b_{qr}, \varphi 1_{kr}, \varphi 2_{kr}, \varphi 3_q, \varphi 4_{ir}) \\ &:= (P^{Y_0} - \kappa\hat{Y})y_r - \sum_{i\in I} P_i^X x_{ir} - \epsilon \sum_{q\in Q} b_{qr} - \sum_{k\in} \varphi 1_{kr} \\ &\times \left(y_r - \hat{\alpha}_k - \sum_{i\in I}\hat{\beta}_{ik}x_{ir} - \sum_{q\in Q}\hat{\gamma}_{qk}b_{qr} - \hat{\mu}\right) \\ &- \sum_{k\in K}\varphi 2_{kr}\left(-\hat{\alpha}_k - \sum_{i\in I}\hat{\beta}_{ik}x_{ir} - \hat{\mu}\right) - \sum_{q\in Q}\varphi 3_q\left(\hat{B}_q - \hat{B}_q^{CAP}\right) \\ &- \sum_{i\in I}\varphi 4_{ir}\left(x_{ir} - X_i^{max}\right), \end{aligned}$$

where $\varphi 1_{kr}, \varphi 2_{kr}, \varphi 3_q$, and $\varphi 4_{ir}$ are the Lagrange multipliers corresponding to each constraint (except the nonnegativity constraint) in model (2).

Based on the first-order conditions of the Lagrangian function and the Karush–Kuhn–Tucker (KKT) conditions (Gabriel et al., 2013; Lee & Johnson, 2015), the MiCP can be formulated as follows, where $x \perp g(x)$ implies $xg(x) = 0$ given a function $g(x)$.

$$\begin{aligned} &0 \le x_{ir} \perp - P_i^X + \sum_{k\in K}\varphi 1_{kr}\hat{\beta}_{ik} + \sum_{k\in K}\varphi 2_{kr}\hat{\beta}_{ik} - \varphi 4_{ir} \le 0, \ \forall i, \ r \\ &0 \le y_r \perp P^{Y_0} - \kappa\hat{Y} - \kappa y_r - \sum_{k\in K}\varphi 1_{kr} \le 0, \ \forall r \\ &0 \le b_{qr} \perp - \epsilon + \sum_{k\in K}\varphi 1_{kr}\hat{\gamma}_{qk} - \varphi 3_q \le 0, \ \forall q, \ r \\ &0 \le \varphi 1_{kr} \perp y_r - \hat{\alpha}_k - \sum_{i\in I}\hat{\beta}_{ik}x_{ir} - \sum_{q\in Q}\hat{\gamma}_{qk}b_{qr} - \hat{\mu} \le 0, \ \forall k, r \\ &0 \le \varphi 2_{kr} \perp - \hat{\alpha}_k - \sum_{i\in I}\hat{\beta}_{ik}x_{ir} - \hat{\mu} \le 0, \ \forall k, r \\ &0 \le \varphi 3_q \perp \hat{B}_q - \hat{B}_q^{CAP} \le 0, \ \forall q \\ &0 \le \varphi 4_{ir} \perp x_{ir} - X_i^{max} \le 0, \ \forall i, r \end{aligned} \quad (3)$$

The Nash equilibrium solution generated from the proposed MiCP (3) exists since the Nash profit function is concave for the maximization problem and PPS forms a convex set.

Theorem 2.1. *The proposed MiCP (3) generates a Nash equilibrium solution $(x_{ir}^*, y_r^*, b_{qr}^*) \in \tilde{T}$, where $\tilde{T}$ is PPS estimated by StoNED frontier model (2) with the cap constraint $\hat{B}_q \le \hat{B}_q^{CAP}$.*

Proof. See Appendix.

Theorem 2.2. *If a profit function is a strictly concave function, continuous and differentiable almost everywhere on $(\boldsymbol{x}_i, \boldsymbol{y}, \boldsymbol{b}_q) \in \tilde{T}$, where $\tilde{T}$ is PPS estimated by the StoNED frontier, then the Nash equilibrium solution found using MiCP (3) is unique if a solution exists for the maximization problem.*

Proof. See Appendix.

Corollary 2.1. *Based on the MiCP (3), the larger the κ value, the lower the desirable output and undesirable output generated and close to zero.*

Proof. See Appendix.

Corollary 2.1 indicates that, based on MiCP (3), the Lagrange multiplier (dual variable) $\varphi 1_{kr}$ is equal to zero when κ is a large enough positive number and y_r is close to zero. This situation suggests that increasing one unit of the right-hand side $\hat{\alpha}_k + \hat{\mu}$ of the first constraint in model (2) does not increase in revenue since the market clearing price is too sensitive with a large κ value.

Corollary 2.2. *The Nash solution generated by MiCP (3) suggests the same efficient benchmark to all the firms.*

Proof. See Appendix.

Corollary 2.2 implies the Nash equilibrium as the only efficient benchmark for all firms and thus there is no need to calculate the average of firm-specific MACs usually generated from different firm-specific efficient benchmarks in previous studies. Using the typical average method could be biased if some benchmark generates an outlier MAC.

3. Nash marginal abatement costs of pollutants

This section introduces the Nash marginal abatement cost (Nash-MAC) for jointly estimating the shadow prices of multiple pollutants simultaneously with respect to the Nash equilibrium benchmark. One profit maximization problem is formulated to investigate the MAC of undesirable output.

We estimate the shadow prices of production technology with desirable outputs (i.e., electricity) and undesirable outputs (i.e., pollutants) by the profit function, which is the profit maximization problem (Lee et al., 2002; Mekaroonreung & Johnson, 2012). Model (4) defines the profit maximization problem with respect to each power plant:

$$\begin{aligned} &\pi(p_y, \boldsymbol{p}_b, \boldsymbol{p}_x) = \max_{y,b,x} p_y y - \boldsymbol{p}_b' \boldsymbol{b} - \boldsymbol{p}_x' \boldsymbol{x} \\ &\text{s.t. } F(\boldsymbol{x}, y, \boldsymbol{b}) = 0, \end{aligned} \tag{4}$$

where p_y, $\boldsymbol{p}_b = (p_{b_1}, \ldots, p_{b_{|Q|}})$, and $\boldsymbol{p}_x = (p_{x_1}, \ldots, p_{x_{|I|}})$ represent the price vectors of the single desirable output, multiple undesirable outputs, and multiple inputs, respectively. $F(\boldsymbol{x}, y, \boldsymbol{b})$ is the transformation function corresponding to a multi-output production technology. Because we want to estimate the shadow prices of the undesirable outputs, we impose the constraint $F(\boldsymbol{x}, y, \boldsymbol{b}) = 0$ so that only the frontier of PPS is considered (i.e. production function). Let τ be a Lagrange multiplier of the constraint. We use the method of Lagrange multipliers to transform the above production to the following Lagrange function:

$$\max_{y,b,x} p_y y - \boldsymbol{p}_b' \boldsymbol{b} - \boldsymbol{p}_x' \boldsymbol{x} + \tau F(\boldsymbol{x}, y, \boldsymbol{b}) \tag{5}$$

To solve the Lagrange function, we apply the first-order conditions (FOCs):

$$\begin{aligned} &p_y + \tau \frac{\partial F(\boldsymbol{x}, y, \boldsymbol{b})}{\partial y} = 0; \; -p_{b_q} + \tau \frac{\partial F(\boldsymbol{x}, y, \boldsymbol{b})}{\partial b_q} = 0; \\ &-p_{x_i} + \tau \frac{\partial F(\boldsymbol{x}, y, \boldsymbol{b})}{\partial x_i} = 0; \; F(\boldsymbol{x}, y, \boldsymbol{b}) = 0 \end{aligned} \tag{6}$$

We derive the shadow price of a pollutant from the FOCs, i.e., Eq. (6) as:

$$\begin{aligned} p_{b_q} &= p_y \left(\frac{\partial F(\boldsymbol{x}, y, \boldsymbol{b})}{\partial b_q} \Big/ \frac{\partial F(\boldsymbol{x}, y, \boldsymbol{b})}{\partial y} \right) \\ &= p_y \left(\frac{\partial y}{\partial b_q} \right) = p_{y_j} \left(\frac{\partial y}{\partial x_i} \Big/ \frac{\partial b_q}{\partial x_i} \right). \end{aligned} \tag{7}$$

In Eq. (7), $\frac{\partial y}{\partial x_i}$ and $\frac{\partial b_q}{\partial x_i}$ are indeed the marginal productivity of electricity and marginal productivity of pollutants by increasing one extra unit of one specific input, respectively. Therefore, we can use the price of electricity, calculated by the inverse demand function regarding the Nash equilibrium, and the ratio ($\frac{\partial y}{\partial x_i} / \frac{\partial b_q}{\partial x_i}$) to estimate the relative MAC (i.e., shadow price) of the pollutants.

Lee and Zhou (2015), who claimed that estimating the shadow prices of each pollutant separately may lead to an underestimation, proposed the directional shadow prices (DSPs) (i.e., directional MACs) of multiple undesirable outputs, given a predetermined direction. Therefore, let $\boldsymbol{g}_x \in \Re_+^{|I|}$, $g_y \in \Re_+$, and $\boldsymbol{g}_b \in \Re_+^{|Q|}$ be the pre-determined directions of the inputs, single desirable output, and undesirable outputs, respectively. Define $g_y + \sum_{q \in Q} g_{b_q} = 1$ for unit simplex (Färe et al., 2013). Let $X_i^{max} = \max\{X_{ik}\}$, $Y^{max} = \max\{Y_k\}$ and $B_q^{max} \max\{B_{qk}\}$ to eliminate the unit of factors. To estimate the DSP, let δ be the decision variable characterizing the slope parameter with respect to the desirable output. Thus, given the direction vector (g_y, g_{b_q}), model (8) is proposed to estimate the directional marginal productivity (DMP) (Lee, 2014) of the desirable output and undesirable outputs by increasing one extra unit of input i^* with respect to the Nash solution (x_i^*, y^*, b_q^*) generated from model (3), as follows:[5]

$$\begin{aligned} &\text{Min } \beta_{i^*} \\ &\text{s.t. } \delta \frac{Y^* - \hat{\mu}}{Y^{max}} = \alpha + \sum_{i \in I} \beta_i \frac{X_i^*}{X_i^{max}} + \sum_{q \in Q} \gamma_q \frac{B_q^*}{B_q^{max}} \\ &Y^{max} \left(\alpha + \sum_{i \in I} \beta_i \frac{X_{ik}}{X_i^{max}} + \sum_{q \in Q} \gamma_q \frac{B_{qk}}{B_q^{max}} \right) \\ &\geq \delta \left(\hat{\alpha}_k + \sum_{i \in I} \hat{\beta}_{ik} X_{ik} + \sum_{q \in Q} \hat{\gamma}_{qk} B_{qk} \right), \; \forall k \\ &Y^{max} \left(\alpha + \sum_{i \in I} \beta_i \frac{X_{ik}}{X_i^{max}} \right) + \delta \hat{\mu} \geq 0, \; \forall k \\ &\delta g_y + \sum_{q \in Q} \gamma_q g_{b_q} = 1 \\ &\beta_i, \gamma_q, \delta \geq 0 \text{ and } \alpha \text{ is free.} \end{aligned} \tag{8}$$

The objective function minimizing β_{i^*} will generate the unique solution which measures the marginal rate approaching from the right side of the Nash efficient benchmark (Lee, 2017). Therefore, given direction $g_y = 1$ and $\sum_{q \in Q} g_{b_q} = 0$, the DMP for electricity $\frac{\partial y}{\partial x_i}$ is calculated by $\frac{g_y Y^{max} \beta_{i^*}}{X_i^{max}}$; while given direction $g_y = 0$ and $\sum_{q \in Q} g_{b_q} = 1$, the DMP for pollutant is calculated by $\frac{\partial b_q}{\partial x_i} = \frac{g_{b_q} B_q^{max} \beta_{i^*}}{X_i^{max}}$, $\forall q$. In particular, the direction vector (g_y, g_{b_q}) can be regarded as the "weightings" between the investigated outputs. The higher the weight, the closer will be the DMP's direction to the output with the higher weight. Then, we use the DMP generated by model (8) for estimating the Nash-MAC by plugging it into Eq. (7). Note that the direction vector here is used to provide a tradeoff among multiple outputs for calculating DMP rather than projecting the inefficient DMU to the frontier.

Theorem 3.1. $\frac{\partial y}{\partial x_i} = \frac{g_y Y^{max} \beta_{i^*}}{X_i^{max}}$ and $\frac{\partial b_q}{\partial x_i} = \frac{g_{b_q} B_q^{max} \beta_{i^*}}{X_i^{max}}$ *based on solving the formulation of model (8) provide the differential characteristics of the StoNED frontier.*

Proof. See Appendix.

[5] After obtaining the Nash solution generated by MiCP (3), the differential characteristics of the production function (i.e., DMP) are not affected by the cap constraint of the undesirable output in model (2). Thus, we only need to consider formulation (1) for developing the model (8).

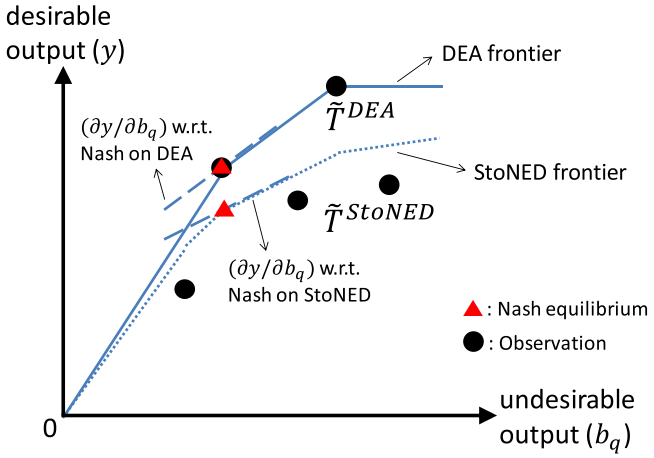

Fig. 1. Nash-MAC estimation on the DEA frontier and StoNED frontier.

Generally, given weak disposability with a concave relationship between desirable output and undesirable output, Corollary 2.1 implies a higher Nash-MAC of undesirable output since a larger κ value generates lower outputs levels with a higher slope $\frac{\partial y}{\partial b_q}$.[6] That is, if the market clearing price is very sensitive with a large κ value, then the competitive relation in an imperfectly competitive market significantly affects a firm's profitability and thus a firm is reluctant to abate the undesirable output with higher MAC. This leads to Corollary 3.1.

Corollary 3.1. *Based on the MiCP (3), the larger the κ value, the lower desirable output and undesirable output generated. Generally, this implies a potential higher Nash-MAC of undesirable output, given a validated direction vector.*

Proof. See Appendix.

Comparing DEA frontier to StoNED frontier, the illustration of Nash-MAC is shown in Fig. 1 with desirable output and undesirable output (for DEA formulations regarding Nash equilibrium and MAC estimation, see Appendix). The DEA frontier covers all the observations based on the minimum extrapolation principle (Fried et al., 2008); however, the StoNED frontier taking into account the composite error term allows some observations located above the frontier. The slopes (or DMP) are investigated with respect to (w.r.t.) Nash equilibrium solution on the two frontiers, respectively.

4. Empirical study

We conduct an empirical study to estimate the Nash-MAC of CO_2, SO_2, and NO_x plant-level coal-fired power plants operating in China in 2013. We focus on the North and the Northeast regions. According to the Natural Resources Defense Council, in China 60% of PM2.5 (airborne particles with a diameter of less than 2.5 microns) is directly generated from coal burning and is especially high in the two regions (Yang, 2014). Thus, these regions show significant abatement potential for emissions reduction and a greater influence on the national goal (Wei, Ni, & Du, 2012). The North region includes Beijing, Tianjin, Hebei, Shanxi, Shandong, and Inner Mongolia, and the Northeast region includes Liaoning, Jilin, and Heilongjiang. In 2013, the State Council of the People's Republic of China issued an Air Pollution Prevention and Control Action Plan to control PM2.5 and reduce the number of smoggy days. The ten-point plan includes limits to pollutant emissions, optimization of energy use, and upgrades to technology. The plan has helped Hebei, Beijing, and Shandong reduce coal consumption by 40, 13, and 20 million tonnes, respectively, compared to their annual consumption in 2012.

For numerical illustration, we assume partial deregulation of electricity market in the North and the Northeast regions. The 33 coal-burning power plants we investigate have nameplate capacity larger than 1000 megawatts, which is large enough to affect the market price.[7] We assume input markets are competitive and thus only consider the imperfectly competitive electricity market for profit maximization. Section 4.1 describes the dataset, and Section 4.2 describes the empirical results of the Nash-MAC estimation via the StoNED and DEA frontiers, respectively. Note that, for comparison, Nash-MAC estimated by DEA frontier, see Appendix or Lee and Zhou (2015).

4.1. Dataset

The balanced plant-level data set comprises 33 coal-burning power plants operating in 2013.[8] We consider coal consumption as the single input, the annual amount of coal-fired electricity generation as the desirable output, and the annual amount of CO_2, SO_2, and NO_x as the three pollutants.[9] The data derive from the China Electric Power Yearbook (CEPP, 2014). The plant-level pollutant emissions are estimated by a plant's proportion of coal consumption multiplied by a province's total emissions and average emission factor (EEA, 2013; IPCC, 2013).[10] The total emissions of the two regions are assigned to the cap with respect to each pollutant, which implies the Nash solution will be generated under this restriction. Table 2 lists the summary statistics.

We also assume a linear price function, i.e., inverse demand function, $P^Y(\hat{Y}) := 4.5 \times 10^7 - 123\hat{Y}$ (unit: CNY$ per 10^8 kilowatt hour) based on China's average on-grid electricity price (wholesale price) CNY$427.01 per megawatt hour and total electricity generation 15,103 (10^8 kilowatt hour) in the two regions in 2013. Note that $\hat{Y} = \sum_k y_k + 10,975$ and the constant $10,975$ represent the electricity generated only by coal-fired plants with a nameplate capacity less than 1 million kW. The price of coal is CNY$590 per tonne. The US Dollar (USD) to Chinese Yuan (CNY) exchange rate is 6.0394.

4.2. Nash MAC Estimation

We estimate the Nash-MAC of CO_2, SO_2 and NO_x via the StoNED with NPF (hereafter StoNEDw/NPF) and DEA with NPF (hereafter DEAw/NPF) and compare the results to the MAC estimation of electric power sectors in previous studies. Due to the three simultaneously emitted pollutants, we estimate the Nash-MAC based on the direction vector $(g^{b_{CO2}}, g^{b_{SO2}}, g^{b_{NOx}}) = (0.048, 0.508, 0.444)$, which is

[6] Note that while the Nash benchmark has lower desirable and undesirable outputs, in some cases the MAC may not become higher since DMP is generated by the projecting hyperplanes constructed by the limited observations using StoNED or DEA.

[7] Different scale size of coal-fired power plants present different emission efficiency; for example, large capacity coal-fired power plants have much better emission efficiency. This issue can be captured by the variable-returns-to-scale property when estimating the production function by StoNED. That is, small plants indicates different shadow price (i.e. slope is more steep on the frontier) from the one estimated by larger plants, as illustrated in Fig. 1. (Note this study only considers the power plant with capacity larger than 1000 mehawatts.)

[8] We exclude Beijing and Jilin because they have no coal-fired power plants with nameplate capacity larger than 1000 mehawatts.

[9] We assume the homogenous quality of coal used in the North and the Northeast regions.

[10] Note that coal-fired power plants consume part of the total coal consumption in all power sectors. However, Coal-fired plant is the major contribution to emission and thus this study focuses on MAC estimation regarding coal-fired plant. This foundation can be extended to other power sectors or industries for further research.

Table 2
Statistics for 33 coal-burning power plants in North and Northeast regions of China in 2013.

Statistics	Coal (10^4 tonne)	Electricity (10^8 kilowatt hour)	CO_2 (10^3 tonne)	SO_2 (10^3 tonne)	NO_x (10^3 tonne)
Avg.	687.641	125.09	14477.16	70.178	67.391
Std. dev.	287.226	59.49	8285.2	25.927	25.446
Max	1767.627	385	46751.08	158.606	153.579
Min	315.882	62	6945.92	36.138	33.147

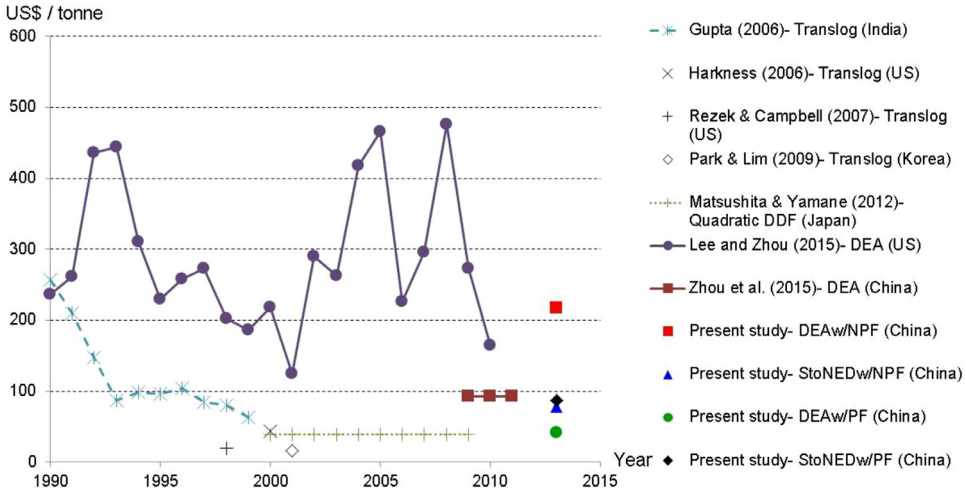

Fig. 2. Comparison of the MAC of CO_2.

Table 3
MAC estimation.

MAC (unit: US$ per tonne)	CO_2	SO_2	NO_x
Upper bound in literature	476.31	8881.34	40335.53
Lower bound in literature	16.12	165.35	450.85
DEAw/NPF	216.81	6038.63	7135.19
StoNEDw/PF	86.80	2417.43	2856.41
StoNEDw/NPF	78.48	2185.83	2582.76
DEAw/PF	41.43	1153.99	1363.54

the direction vector suggested by Lee and Zhou (2015). We also compare with the MAC estimated by a typical profit function (PF) with exogenous price (Zofio et al., 2013), i.e., StoNED with PF (hereafter StoNEDw/PF) and DEA with PF (hereafter DEAw/PF), to investigate whether the model choice and endogenous price have significant impact on the MAC.

Table 3, Figs. 2–4 illustrate the comparisons of the MACs of CO_2, SO_2, and NO_x from the studies in 1980–2015. The results show a consistent result of MACs regarding the three types of pollutants, respectively; as a whole, the descendent sequence of MAC estimates is DEAw/NPF, StoNEDw/PF, StoNEDw/NPF, and DEAw/PF. This implies that a typical method DEAw/PF underestimates the MAC due to an invalid assumption of the exogenous price; however, the DEAw/NPF gives a higher value since the composite effect from the endogenous price and the outlier-sensitive DEA. On the other side, a small gap between StoNEDw/PF and StoNEDw/NPF implies that the relatively smooth StoNED frontier may weaken the effect of endogenous price and provide the more robust MAC estimates under different price assumptions. Thus, this study proposes a range of MAC estimate between StoNEDw/NPF and DEAw/NPF in an imperfectly competitive market.

Now, we compare the proposed Nash-MAC range (i.e., from StoNEDw/NPF and DEAw/NPF) with previous studies and provide the managerial insights.[11] Fig. 2 shows that the proposed range from US$78.48 to 216.81 per tonne provides a moderate MAC of CO_2 in literature, but in 2013, the average allowance price of carbon is US$8.78 per tonne in Beijing and US$5.18 per tonne in Tianjin, respectively. Thus, the two regions should purchase emission allowances of CO_2 from the market to address the carbon emission.

Fig. 3 shows a higher MAC range of SO_2 from US$2185.83 to 6038.63 per tonne, whose average is larger than the average MAC US$3492 by Lee and Zhou (2015) and US$1072 by Mekaroonreung and Johnson (2012). Thus, the MAC result of SO_2 imply that the abatement technology and policy of SO_2 in the coal-fired power industry is urgent in the North and Northeast regions of China; nevertheless, a higher MAC range implies that the development of abatement technology for reducing SO_2 is not affordable. In the short run, the two regions should purchase emission allowances of SO_2 from the market if the allowance price is much lower than the MAC.[12] In fact, until 2014 China's government developed a series of policies to control SO_2 emission and that desulfurization equipment be installed in both existing and new plants. The policies make it more difficult to reduce SO_2 emission using only engineering techniques. The result shows that the plants with extra allowances remain reluctant to sell their allowances, making it difficult for a plant lacking allowances to meet the regulation requirement by purchasing them from plants with extra allowances.

Fig. 4 shows that the proposed range from US$2582.76 to 7135.19 per tonne provides a relatively lower MAC of NO_x, below

[11] Although different regions may have different policies and lead to different MAC results, the comparison is still worthy to be mentioned since the coal-fired power plant is the major emission source in all world-wide areas.

[12] In general, higher MAC means that it's beneficial to a firm if it applies any methods with a cost less than the MAC to reduce the emission, no matter whether it is through allowance purchase, or abatement technology deployment. In fact, the production function estimation implicitly modeled the "technology" (including the existing abatement technologies). Thus, this study suggests purchasing allowance to address the unaffordable abatement technology development.

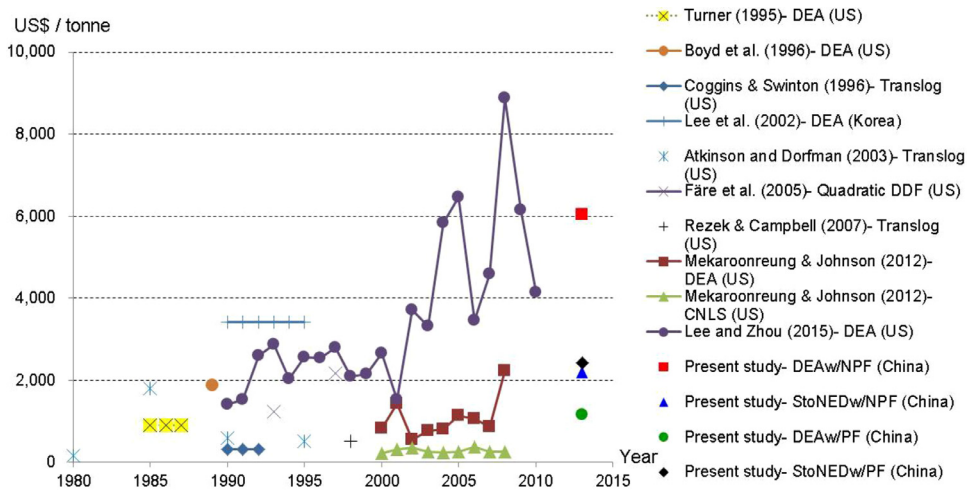

Fig. 3. Comparison of the estimated shadow prices of SO_2.

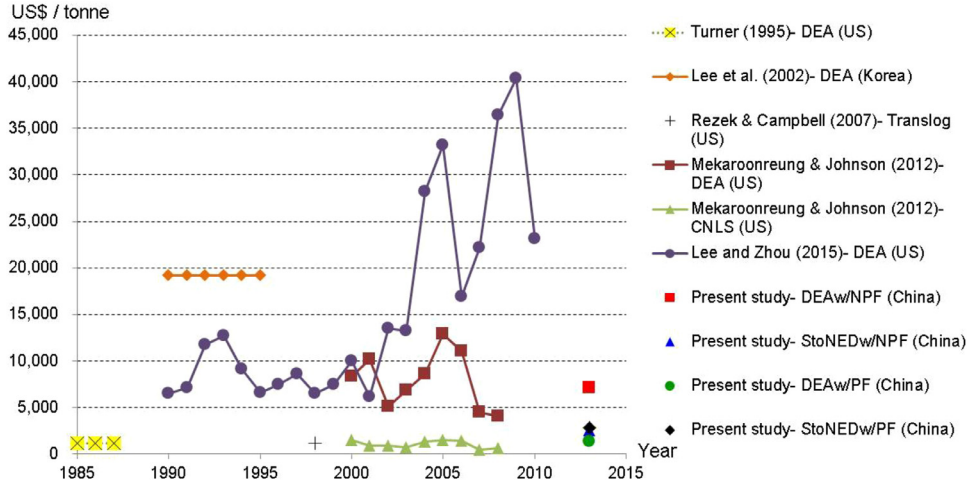

Fig. 4. Comparison of the estimated shadow prices of NO_x.

an average of around US$7929 suggested by Mekaroonreung and Johnson (2012) and much lower than US$15572 suggested by Lee and Zhou (2015). The result implies plants are encouraged to invest in the development of NO_x abatement techniques. While the allowance price is lower than the MAC, in the long run, when carbon regulation becomes more stringent, the MAC and allowance price are likely to rise.

In addition, policy-makers or managers can depend on the MAC range to measure the risk when developing appropriate abatement technology. For example, we can observe that the MAC variation of NO_x is small, whereas the MAC variation of SO_2 is large. This implies that policy-makers or managers may take less risk to invest in NO_x abatement technology even though the market structure migrates in the period of technology development. In fact, as mentioned above, it is worthwhile to develop the NO_x abatement technology due to a historical relatively low MAC, whereas developing the advanced SO_2 abatement technology is not encouraged since China has already made efforts to reduce this pollutant in the past two decades.

We also derive the following theoretical insights. First, DEAw/PF underestimates the MAC due to an invalid assumption of the exogenous price. Second, a large gap between DEAw/NPF and DEAw/PF implies that the endogenous price factor significantly affects the MAC estimation in DEA. Thus, clarifying the market structure and using the justified price assumption, i.e., exogenous or endogenous, are critical in studying the MAC estimated by DEA frontier, in particular, in an imperfectly competitive energy market. Third, we theorize that the choice of DEA model or StoNED model will affect the Nash-MAC of emission (see the gap between DEAw/PF & StoNEDw/PF or the gap between DEAw/NPF & StoNEDw/NPF). The main reason is that StoNED considers the composite error term including noise and inefficiency and estimates a production function more robust than DEA, i.e., reducing the outlier effect which DEA cannot address well. In particular, the endogenous price factor enlarges the MAC gap (see the gap between DEAw/NPF and StoNEDw/NPF). Thus, the model selection (DEA or StoNED) plays an important role in MAC estimation and correlates with the price assumption. Fourth, as mentioned, a small MAC gap between StoNEDw/PF and StoNEDw/NPF validates that the StoNED frontier provides a "smoother"[13] shape of the frontier than the DEA frontier. In other words, the StoNED frontier provides more robust MAC estimates no matter of exogenous or endogenous price assumptions. The result suggests using the StoNED approach for a moderate MAC estimation (since DEAw/PF < StoNEDw/NPF < StoNEDw/PF < DEAw/NPF) when there

[13] Note that, in fact, the StoNED frontier is a non-smooth piece-wise linear frontier with a small variation among the slope coefficients of different hyperplanes.

is a larger or unclear variation of the collected dataset, or unsure price assumptions in an unrecognizable market structure.

5. Conclusion

Knowing the MAC (or shadow price) of pollutants provides useful environmental policy guidelines, such as the allowance price in emissions trading markets and the penalty rates for emissions. A typical profit function used in the DEA frontier implicitly assumes a perfectly competitive market and underestimates the MAC (i.e. DEAw/PF), whereas this paper proposes a Nash profit function to identify the Nash equilibrium on the StoNED frontier in an imperfectly competitive market and the proposed MAC range between StoNEDw/NPF and DEAw/NPF corrects the issue. An empirical study estimates the Nash-MAC of CO_2, SO_2, and NO_x in coal-fired power plants operating in the North and the Northeast regions of China in 2013.

In fact, plants participating in the ETS should have purchased or reserved the allowances in 2013 since the CO_2 and SO_2 allowances are likely to be undervalued. In particular, an excess allocation of carbon allowances (e.g., in 2013, the government of Beijing allocated 50 million tonnes of allowances, but only 1.55 million tonnes of allowance were traded before the closing date for paying the allowances for 2015) and a slight penalty imposed for violating the allocated quota (e.g., around three times the allowance price in 2013) undermined the ETS. Setting a more stringent standard to control these emissions should improve China's carbon market. In 2013, China's State Council issued its Air Pollution Prevention and Control Action Plan to control coal consumption and set the target of coal consumption at less than 65% of total energy sectors in 2017. China's recent coal over-supply and electricity over-surplus in the coal-fired power plant sector provides a chance to restructure its energy systems in favor of renewable energy.

This paper addresses four key issues in existing models. First, we consider the endogenous price issue in an imperfectly competitive market and suggest the Nash solution as a benchmark addressing the direction-dependent shadow price in the published literature. Second, the Nash solution is identical to all firms and the average (or weighted average) of MAC is unnecessary. Third, we use a stochastic semi-nonparametric frontier which avoids the functional form assumption and accounts for noise to generate a robust Nash solution, and thereby a moderate MAC. Fourth, using the DMP to estimate the MACs of multiple pollutants simultaneously avoids underestimating MACs.

We suggest that the development of applying the Nash-MAC models to the latest data after the implementation of the pollution standards, e.g., jointly computing the shadow price of other emissions (e.g. mercury), will provide useful data to support regulatory policy. Future research should investigate some uncertain issues, such as the volatility of electricity demand (Lee, 2015; Lee & Johnson, 2014) and price, and consider other parties, such as the transmission provider and the trading regulator, in the power network system for the equilibrium problem.

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Supplementary materials

Supplementary material associated with this article can be found, in the online version, at doi:10.1016/j.ejor.2018.08.016.

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